EDA (estimation distribution algorithms)

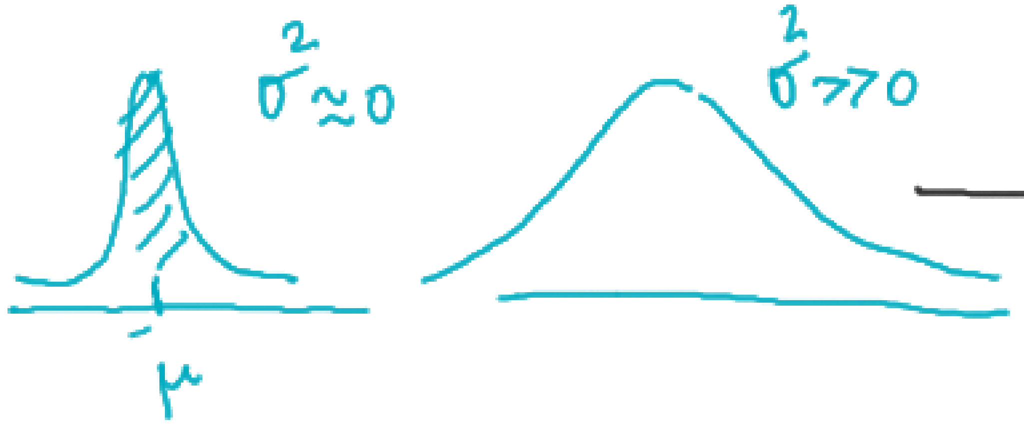
Repaso de estadística:

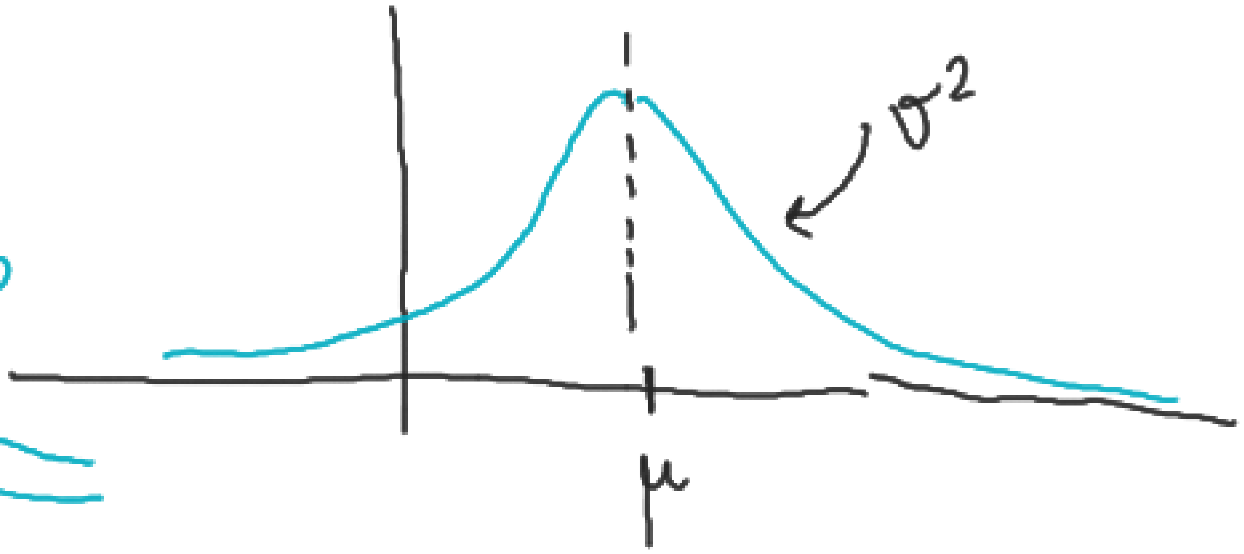
$N(\mu, \sigma^2)$

media varianza

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

función de densidad de $N(\mu, \sigma^2)$

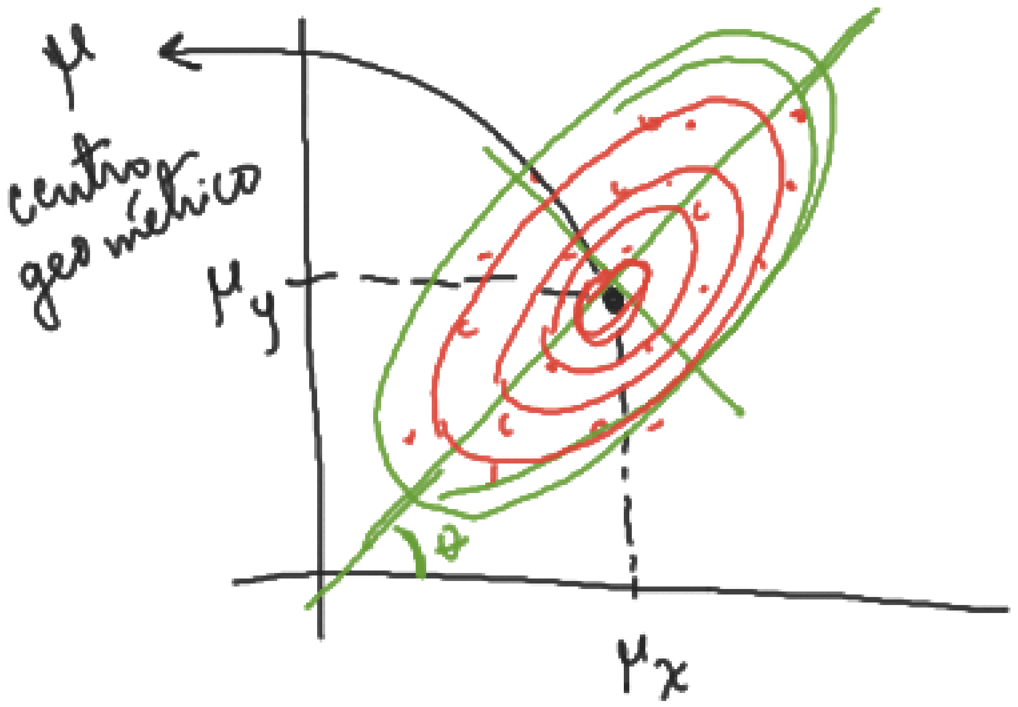




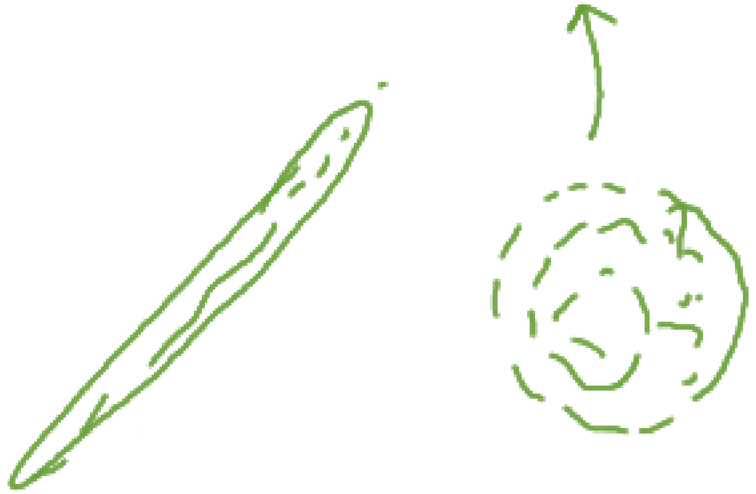
En $\mathbb{R}^2$:

$\mathbf{x} = (x, y)$ $\quad\quad \mathbf{x} \sim N_2(\mu, \Sigma)$

$\mu = (\mu_x, \mu_y) \in \mathbb{R}^2$ $\quad\quad \Sigma = \text{covarianza} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$

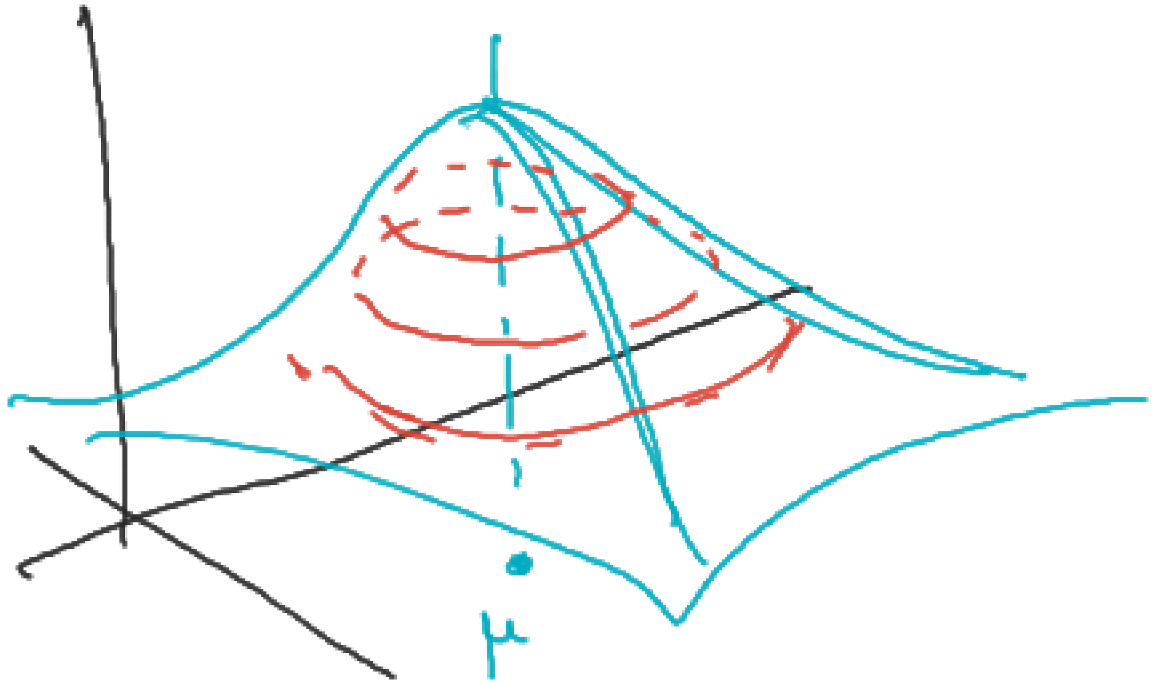


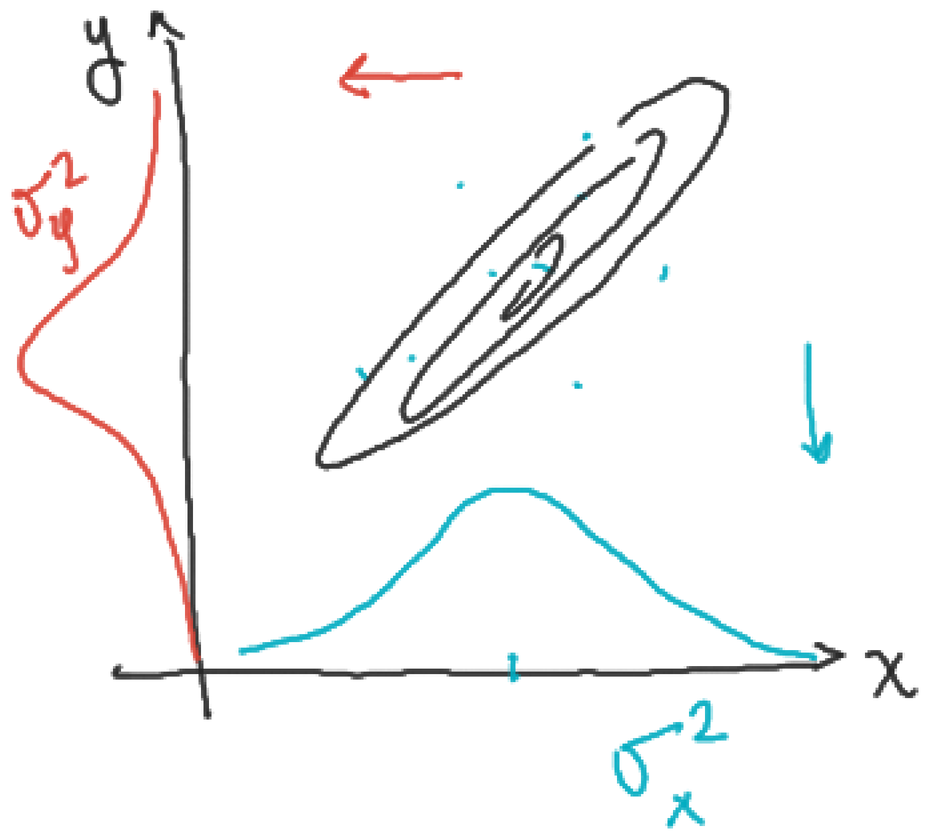
$\bullet \sim N_2(\mu, \Sigma)$ $\qquad \mu = (\mu_x, \mu_y)$



$$f(\mathbf{x}) = f(x,y) = \left(\frac{1}{\sqrt{2\pi}} \frac{1}{|\Sigma|^{1/2}} \right)^2 e^{-(x-\mu)^T \Sigma^{-1} (x-\mu)/2}$$

$\mathbf{x} = (x,y)$





$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

σ_{xy} = covarianza de x, y

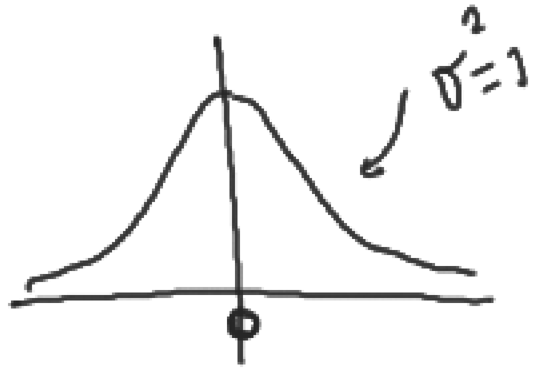
$$= \frac{1}{n}\sum_{i=1}^{n} (x_i-\mu_x)(y_i-\mu_y)$$

$$= \mathbb{E}\big((X-\mathbb{E}X)(Y-\mathbb{E}Y)\big)$$

Distribución normal estándar

$N(0,1)$

$\mu=0$
$\sigma^2=1$



$$Z \sim N(0,1)$$

$$X = \mu + \sigma Z \sim N(\mu,\sigma^2)$$

Normal Estándar 2D

$\sigma_{xy} = 0$

$Z \sim N((0,0), I_{2\times 2})$

$I_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

x, y son independientes

$\mu = (0,0)$

$\Sigma = I_{2\times 2}$

¿Cómo hacemos para construir una $N_2(\mu, \Sigma)$?

1D: $X = \mu + \underbrace{\sigma}_{\sigma = \sqrt{\sigma^2}} Z$

2D: $X = \mu + \underbrace{\sqrt{\Sigma}}_{?} Z$

Descomposición espectral ó Descomposición SVD (singular value decomp.)

PCA

$\Sigma = U S V^T$

$$\Sigma = \left[\quad\right]_{2\times 2} = \underbrace{\left[\quad\right]}_{U}\underbrace{\begin{bmatrix} s_1 & \\ & s_2 \end{bmatrix}}_{S}\underbrace{\left[\quad\right]}_{U^T}$$

$$\Sigma^{1/2} = \underbrace{\begin{bmatrix} \sqrt{s_1} & \\ & \sqrt{s_2} \end{bmatrix}}_{S^{1/2}}\underbrace{\left[\quad\right]}_{U^T}$$

$$\underbrace{\Sigma^{1/2}\cdot\Sigma^{1/2}}_{\text{raíz}} = \Sigma$$

$$\boxed{X = \mu + Z\Sigma^{1/2}}$$

$$X \sim N_2(\mu, \Sigma).$$