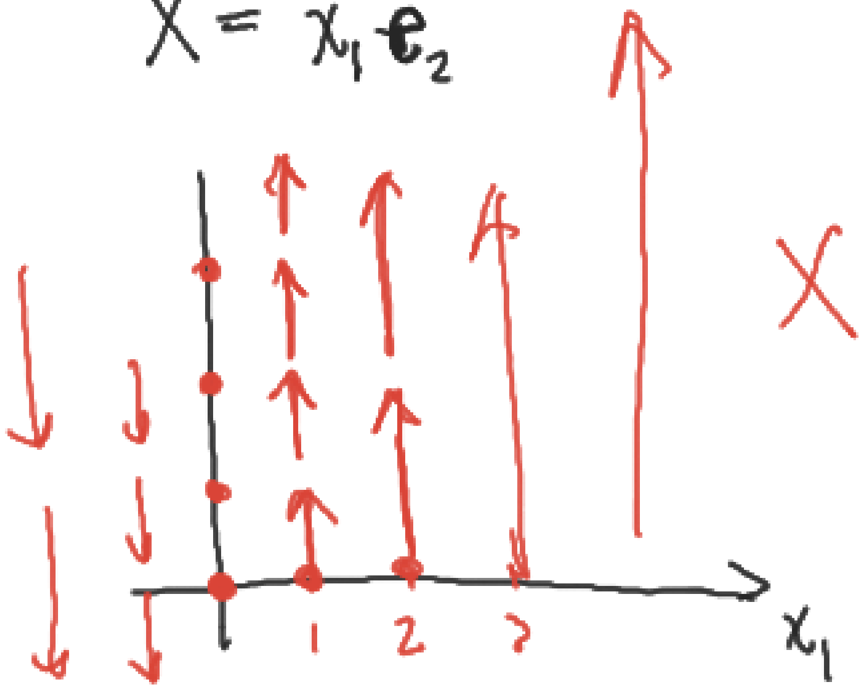
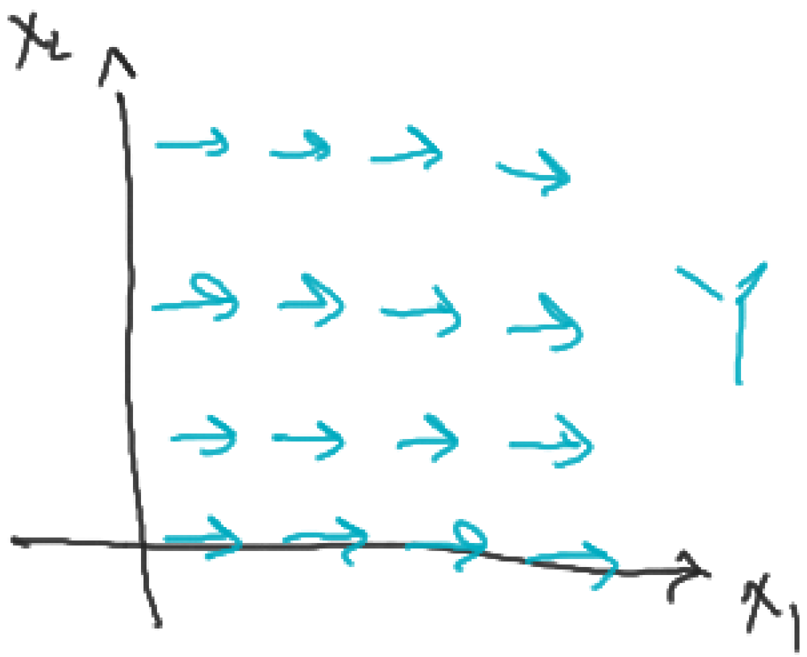
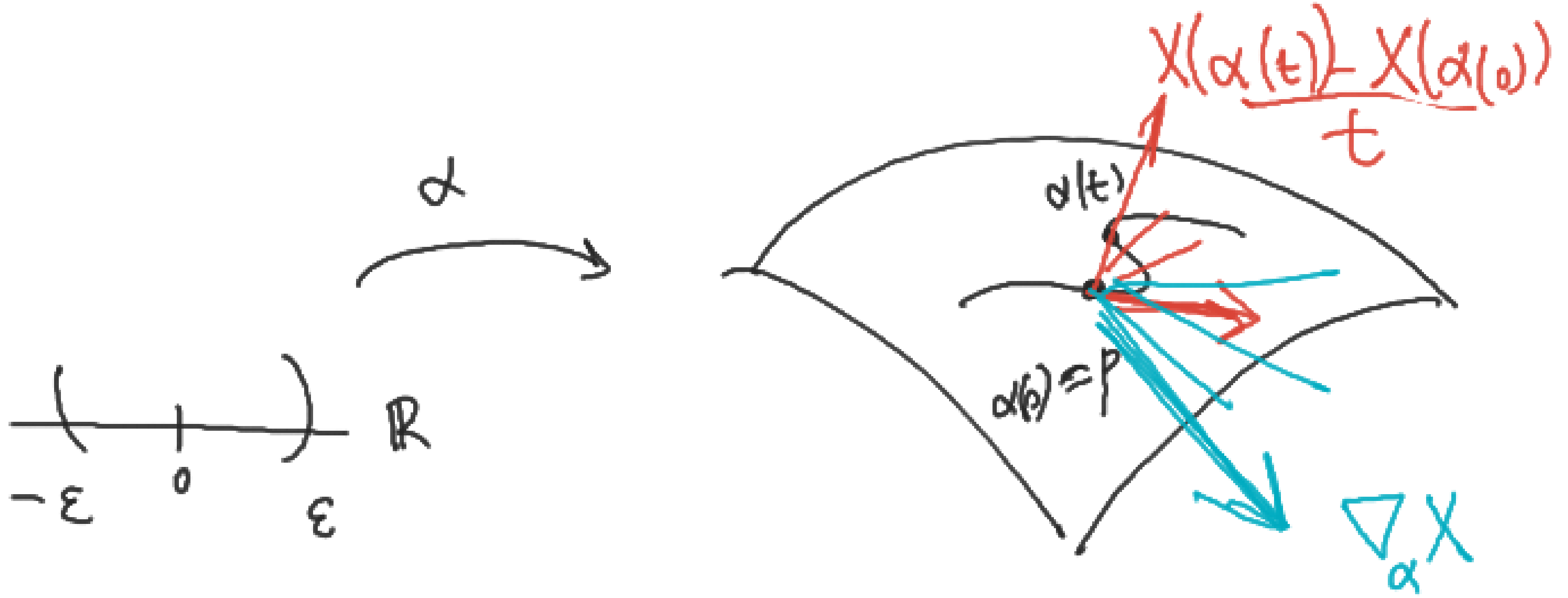
$X = x_1 e_2$





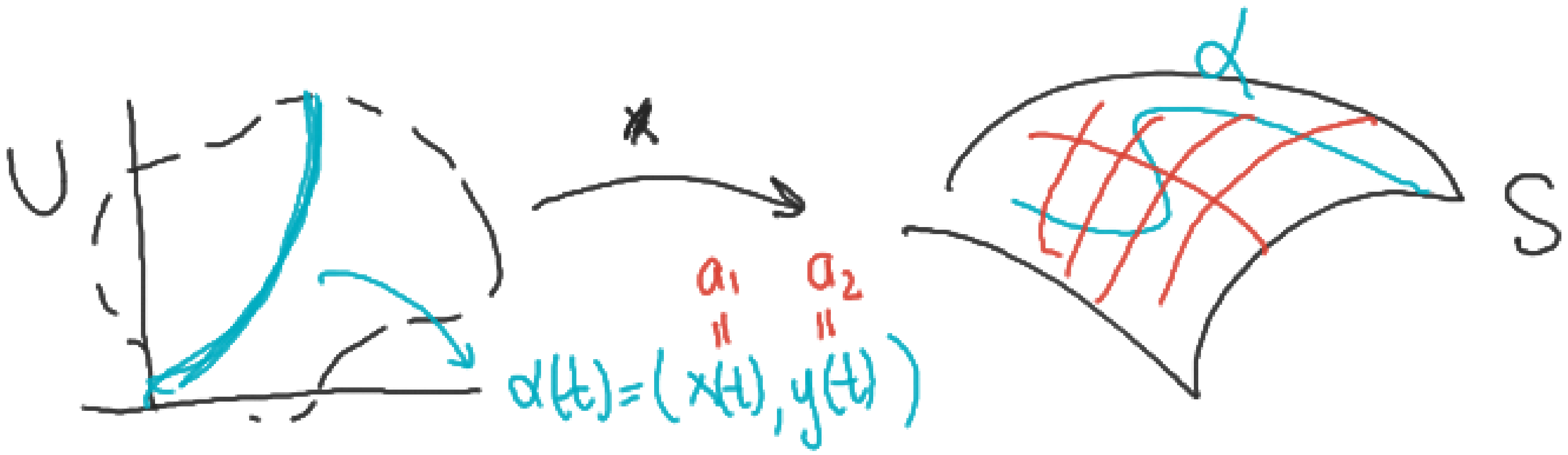

α
$\mathbb{R}$
$-\varepsilon$
0
ε
$\alpha(t)$
$\alpha(0) = p$
$\frac{X(\alpha(t)) - X(\alpha(0))}{t}$
$\nabla_{\alpha} X$

En superficies:

$$a_k'' + \Gamma_{ij}^{k} (a^i)'(a^j)' = 0 \qquad i,j,k = 1,2$$

$$a_1'' + \Gamma_{11}^{1} (a_1)'(a_1)' + \Gamma_{12}^{1} (a_1)'(a_2)' + \Gamma_{21}^{1} (a_2)'(a_1)' + \Gamma_{22}^{1} (a_2)'(a_2)' = 0$$

$$a_2'' + \Gamma_{11}^{2} (a_1)'(a_1)' + \Gamma_{12}^{2} (a_1)'(a_2)' + \Gamma_{21}^{2} (a_2)'(a_1)' + \Gamma_{22}^{2} (a_2)'(a_2)' = 0$$



$$x_1 = x \qquad x_2 = y$$

$$x'' + \sum_{i,j=1}^{2} \Gamma_{ij}^{1} \underline{(x_i)'(x_j)'} = 0$$

$$y'' + \sum \Gamma_{ij}^{2} \underline{(x_i)'(x_j)'} = 0 .$$

sistema de EDO no-lineal

Ej: (Plano)

$$x_k'' + \sum_{i,j} \cancelto{0}{\Gamma^k_{ij}}\, x_i' x_j' = 0 \quad \Longrightarrow \quad x_k'' = 0, \qquad k = 1,2.$$

$$\Rightarrow \begin{cases} x'' = 0 \\ y'' = 0 \end{cases} \Rightarrow \quad \begin{aligned} x(t) &= x_0 + x_1 t \qquad & x_0, x_1 \in \mathbb{R} \\ y(t) &= y_0 + y_1 t \qquad & y_0, y_1 \in \mathbb{R} \end{aligned}$$

$$\alpha(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} x_1 t \\ y_1 t \end{pmatrix} \qquad p_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

$$= \underline{p_0 + \mathbf{v} t} \qquad t \in \mathbb{R}.$$