Cociente de Rayleigh

$$c = \frac{x^T A x}{x^T x}$$

A simétrica y definida positiva

$x \neq 0$

<u>Def</u>: A es definida positiva si $x^T A x > 0, \ \forall x \neq 0$.

<u>Matrices de rango 1</u>

$$M = \begin{bmatrix} \boxed{v_1} \,|\, v_2 \,|\, \dots \,|\, v_d \end{bmatrix}, \quad v_j \in \mathbb{R}^d$$

$\text{rank}(M) = 1 = \dim(\text{Im}\, M)$

$\text{Im}\, M = \langle v_1, v_2, \dots, v_d \rangle$

$\Rightarrow$ todos los v_j son múltiplos

$$M = \begin{bmatrix} \boxed{r_1} \\ \hline r_2 \\ \vdots \\ \hline r_d \end{bmatrix}$$

Caso particular: $u, v \in \mathbb{R}^d \quad (u, v \neq 0)$

$$u = \begin{bmatrix} u_1 \\ \vdots \\ u_d \end{bmatrix}$$

$$v^T = \begin{bmatrix} v_1 & \cdots & v_d \end{bmatrix}$$

$$\boxed{M = uv^T} = \underset{(d \times 1)}{\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_d \end{bmatrix}} \underset{(1 \times d)}{\begin{bmatrix} v_1 & v_2 & \cdots & v_d \end{bmatrix}}$$

$$= \begin{bmatrix} u_1 v_1 & u_1 v_2 & \cdots & u_1 v_d \\ u_2 v_1 & u_2 v_2 & \cdots & u_2 v_d \\ \vdots & & & \vdots \\ u_d v_1 & u_d v_2 & \cdots & u_d v_d \end{bmatrix}$$

$$F_2 = \frac{u_2}{u_1} F_1$$

$$C_2 = \frac{v_2}{v_1} C_1$$

Teorema Espectral: $A \in \mathbb{R}^{d \times d}$ simétrica

$$A = U \Lambda U^T = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_d \\ | & | & & | \end{bmatrix} \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_d \end{bmatrix} \begin{bmatrix} u_1^T \\ u_2^T \\ \vdots \\ u_d^T \end{bmatrix}$$

$$= \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_d \\ | & | & & | \end{bmatrix} \begin{bmatrix} \lambda_1 u_1^T \\ \lambda_2 u_2^T \\ \vdots \\ \lambda_d u_d^T \end{bmatrix}$$

$$= u_1 \cdot \lambda_1 u_1^T + u_2 \cdot \lambda_2 u_2^T + \dots + u_d \cdot \lambda_d u_d^T = \sum_{i=1}^{d} \lambda_i \underline{u_i u_i^T}.$$

$$A = \boxed{\lambda_1 u_1 u_1^T} + \boxed{\lambda_2 u_2 u_2^T} + \cdots \cancel{+} \cancel{\lambda_d u_d u_d^T}.$$

k

$+ \qquad = \qquad$ importante $\lambda_1 > \lambda_2 \cdots > 0$

¿Cómo hallar el subespacio r-dimensional, sobre el cual proyecto los datos para maximizar $Var(l_i^T X)$?

- $\mathbb{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1d} \\ \vdots & \vdots & & \vdots \\ x_{n1} & x_{n2} & & x_{nd} \end{bmatrix}$ y <u>centramos</u> los datos

- $Cov(X) = \mathbb{X}^T \mathbb{X} \in \mathbb{R}^{d \times d}$

- Descomposición espectral a $Cov(X)$

$$Cov(X) = \begin{bmatrix} u_1 & u_2 & \cdots & u_d \end{bmatrix} \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_d \end{bmatrix} \begin{bmatrix} u_1 & u_2 & \cdots & u_d \end{bmatrix}^T$$

$$\uparrow \uparrow \uparrow$$

Tomamos $\langle u_1, u_2, \ldots, u_r \rangle$. $\quad 1 \leq r \leq d. \quad \lambda_1 \geq \lambda_2 \ldots \geq 0$

$$A = \begin{bmatrix} a_{11} \cdots a_{1m} \\ \vdots \\ a_{m1} \cdots a_{nm} \end{bmatrix} = [a_{ij}]$$

$$\|A\|_F^2 = \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij}^2 \qquad \text{norma de Frobenius}$$